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A simplified computational model for hydrodynamic studies on dilute phase pneumatic transport in vertical riser

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A simple theoretical model has been developed to analyze the hydrodynamics of gas flow laden with low concentration of particulate solids through the vertical riser. The model has been systematically developed following the flow dynamics similar to the work of Dhodapkar *et al*. resulting in an algebraic equation of differential height as a function of differential particle velocity. The individual pressure drop components have been also expressed as a function of differential particle velocity. Based on the said model, one FORTRAN simulation program has been developed to estimate the various parameters like static pressure drop, gas and particle velocities and particle Reynolds number profiles along the vertical riser by integration. The theoretical pressure drop data has been compared with the experimental data of earlier experiments conducted by Dzido *et al.* and also Sarkar to validate the model and the deviations have been also analyzed. Deviation of the experimental static pressure data from the estimated values near the solid entry point has been justified and the proposed model has been verified after minor adjustment. The various hydrodynamic parameters computed by the simulation program have been also analyzed.

Keywords: Vertical riser, Flow dynamics, Pressure loss, Particulate solids, Simulation

A number of theoretical models were presented during last few decades by earlier researchers since pneumatic transport has been used for transportation of particulate solids with air or gas in different industrial applications. The basic purpose of the earlier models was to study the pressure drop criteria across the pneumatic transport unit in order to optimize the pumping energy compared with other conventional particulate solid transportation methods. Majority of the models were based on the basic equations of transport phenomena related to hydrodynamics of such flow systems and solving the differential equations by standard numerical techniques using different software. But a few earlier authors also dealt with the hydrodynamics of such systems, based on the simple dynamic equations related to the movement of solid particles by the drag force applied on the particle by gas, against the resistive gravitational force, frictional force between the particles, and between the particle and inner wall of the transport tube. Those approaches were relatively simple with some deviations also. The present theoretical model used this simple technique.

Numerous experimental studies were conducted specifically on vertical pneumatic transports, which are available concisely in review literatures, out of which, a few might be mentioned in the present context. Cramp and Priestley[2] were the pioneer investigators in pneumatic transport and developed an equation for estimation of pressure drop assuming the wall-particle friction to be negligible**.** Similarly, Hariu and Molsted[7] also analyzed the pressure drop criteria. According to them the pressure-drop for flow of gas-particle suspension was simply regarded as the sum of individual pressure-drops due to carrier gas and solids like, $\Delta P_t=\Delta P_s+\Delta P_g$. Horio *et al.*[8] considered the pressure drop in the riser as the algebraic summation of pressure drop for suspending the solids, particle-wall friction loss, and acceleration loss. Rautiainen and Stewart[1] conducted experimental studies on vertical pneumatic conveyers using a one–dimensional equation and experimental techniques to provide a comprehensive description of vertical gas-solid two phase flow. The results from non-accelerating flow experiments conducted with a riser tube of bore 192 mm and height 16.2 m using spherical glass beads of average diameter 64 micrometer were presented. They found that the frictional pressure drop was recognized to be an important component of the total pressure gradient in the riser. Ferreira and Narimatsu[4] worked on vertical pneumatic conveyers in dilute and dense phase flows. They studied the effects of particle size and density on the fluid dynamic behavior of vertical gas-solid transport of group D particles in a 53.4 mm diameter transport tube. They observed that the experimental curves of pressure gradient versus air velocity presented a minimum pressure gradient point, which is associated with a change in the flow regime from dense to dilute phase. An empirical equation was fitted for predicting transition velocity between dense and dilute phase for the transport of glass spheres. Dzido and Pallicka *et al.*[5] studied the investigations of the acceleration region in vertical pneumatic conveyers. They have shown that pressure drop in the acceleration region can be predicted using the uniform flow model if the proper value of the initial solid velocity is known. The Merson method was applied to obtain a numerical solution of the set of governing equations. This method offers the possibility of estimation of integration error and hence optimization of step length and reduction of computational time. Grbavcic *et al.*[6] formulated a one-dimensional model of accelerating turbulent gas-solid dilute phase flow of coarse particles and experimentally verified by measuring pressure distribution along the transport tube. A critical review of earlier literatures on vertical pneumatic conveying was done by the present authors, Anandhakrishnan Sarkar *et al.*[14] which may be also referred in this context.

The earlier researchers considered gas-solid flow models in pneumatic conveying which has been broadly classified by Klinzing[9] into two namely, uniform flow model in case of vertical dilute phase and core-annulus flow model in dense phase.

Present model

The present computational model is based on the uniform flow model following the simple approach which is restricted to dilute phase transport only. This model can estimate not only the pressure drop but also other parameters like, velocity of the particles and the transporting fluid, particle Reynolds number, void-fraction etc. These parameters were significant in case of transport phenomena like mass and heat transport between the gas and solid which occurs in the field of applications other than the simple pneumatic transport. The various hydrodynamic parameters were computed using the simulation based on present model. The static pressure computed by the present model was compared with the experimental results of other experimenters.

Development of a computational model for estimation of hydrodynamic parameters at different riser heights

An algorithm has been developed for computing the profiles of the variables like, pressure drop, particle velocity, slip velocity, gas and particle Reynolds number and void fraction along the height. For vertical dilute phase pneumatic transport, the underlying assumptions were considered.

1. Particles are spherical in shape and have equal average size, which do not alter during transport.
2. The particles are uniformly distributed throughout the cross section of the vertical conduit immediately after introduction through the feed point, without any radial variation in the solid concentration which persists along the vertical riser. The buoyant force due to the fluid medium (gaseous) is negligible in comparison with the gravitational force acting on the particle.
3. The solid particles experience only three types of forces like, upward fluid drag exerted by air, downward gravitational force due to the weight of solids and average solid frictional force from the instant of release into the upward flowing air,.
4. The physical and transport properties of the fluid medium (air) such as density and viscosity remains constant, considering the system to be isothermal and pressure variation is low.
5. Based on the above underlying assumptions, the final equations have been deduced and used to develop the computational simulation program.

The frictional force applied on the particles is assumed to be governed by Konno-Saito[10] relationship. So, for a riser diameter of d_t having particle velocity, u_p

$F_S = 2\ f_s u_p^2 \frac{dh}{d_t}$ where, friction factor, $f_s = k / \{u_p / (gd_t)^{1/2}\}$.

Wen and Galli[13] pointed out that in an assembly of particles the void fraction is to be considered and drag coefficient, C_d has to be replaced by C_{dm} where, $C_{dm} = C_d\varepsilon^{-4.7}$ which may be significant near the particle feeding point.

Considering the aforesaid relationships, the drag force, F_d applied on a single solid particle may be written as

$$F_d = \frac{1}{2} C_{dm}\, \rho_g (u_a - u_p)^2] \frac{\pi d_p^2}{4}$$

$$= \frac{\pi}{8} C_d (\varepsilon^{-4.7})\, \rho_g\, d_p^2 (u_a - u_p)^2.$$

Therefore, net force applied on the particles in the volume of differential thin lamina of height dh = drag force - gravitational force - frictional force

Or, $\{\left(\frac{\pi d_t^2}{4}\right)$ (dh) (1-ε) $\rho_p\}(du_p / dt)$

$= \{(\frac{\pi d_t^2}{4})$ dh (1-ε)$\{1/\pi dp^3/6)$ $(\frac{\pi}{8})$ Cd $(\varepsilon^{-4.7})$ ρ_g d_p^2 $(ua\text{-}up)^2\}$ - $\{(\frac{\pi d_t^2}{4})$ dh (1-ε) ρp g}

$-\{2f_s\, u_p^2\,(1\text{-}\varepsilon)\,\rho p\,(\frac{dh}{dt})\,(\frac{\pi d_t^2}{4})\}$ … (1)

The drag co-efficient, C_d attains a constant value of 0.44 in the Newtonian regime and is equal to $(24/Re_p)$ in the laminar region. A generalized expression for C_d has been suggested by Morsei and Alexander[11] as follows

$C_d = (k_1/Re_p) + (k_2/Re_p^2) + k_3$ … (2)

where, $Re_p = \frac{(u_a - u_p)(dp)(\rho g)}{\mu}$

and k_1, k_2, k_3 are empirical constants.
After simplification, one can get,

$$dh = \frac{(u_p^4)\,du_p}{(\alpha u_p^4 + \beta u_p^3 + \gamma u_p^2 + \delta u_p + \psi)} \quad \ldots(3)$$

In the above polynomial function, α, β, γ, δ and ψ in the denominator, are all functions of empirical constants k_1, k_2 and k_3, gas and solid flow rates and also the physical properties of solid particles and riser diameter, all of which are constants for a particular case. The equation (3) may be used in the simulation for estimation of particle velocity profile along vertical riser.

Estimation of pressure drop and void fraction profiles in the present model

The total pressure drop profile within the differential height dh in the vertical riser has been computed considering the total pressure drop as the algebraic summation of five different pressure loss components which was shown by Horio *et al.*[8]

$dp_t = dp_{sg} + dp_{fg} + dp_{ss} + dp_{fs} + dp_{as}$ … (4)

where, dp_t = total pressure drop across the element dh. Each pressure drop component has been expressed in terms of gas and average particle velocity within differential height, dh.

Pressure drop due to static column of fluid,

$dp_{sg} = \rho_g\, \epsilon_g g dh$ … (5)

Pressure drop for the fluid friction with the wall,

$dp_{fg} = (2f_g\rho_g u_a^2\varepsilon/dt)$ dh … (6)

Pressure drop due to static column of solid particles,

$dp_{ss} = \rho_p(1\text{-}\varepsilon)$ g dh … (7)

Pressure drop as a result of solid friction with the tube wall,

$dp_{fs} = (2f_s\rho_p(1\text{-}\varepsilon)u_p^2/dt)$ dh … (8)

and, pressure drop due to acceleration of the solid particles,

$dp_{as} = 1/2\, \rho_p(1\text{-}\varepsilon)[(u_p + du_p)^2 - u_p^2)]$ … (9)

Since dup is infinitesimally small, (dup)2 is negligible and one can write,

$dp_{as} = \rho_p(1\text{-}\varepsilon)\, u_p\, du_p$ … (10)

Thus, after integration the individual pressure components and the total pressure drop (TDELP) can be evaluated theoretically at different heights to generate the pressure drop profiles along riser height at various gas velocity, physical properties and riser diameter. During computation in the program, the total riser height is divided into differential smaller heights and assuming uniform condition within the circular lamina considering radial uniformity.

One of the computed static pressure profile along the vertical riser is compared with the experimental data of Dzido *et al.*[5] with its experimental conditions and also that of Sarkar[12]. The details of the solid particles and conveying air are shown in Table 1. The same set of data was used in the computational simulation as input data for validation of the present model.

Results and Discussion

Validation of simulation model with experimental results of Dzido *et al.* and Sarkar

The hydrodynamic computational model presented by the present authors for the vertical gas solid flow

Table 1 — Physical parameters of the particles and gas samples used for experiments

S.No	Material used	Particle size (m)	Solids feed rate (kg/s)	Air velocity (m/s)	Solid loading ratio (SLR)
1*	Sand	3.46×10^{-4}	128.6	10.13	3.12
2#	Sago beads	1.55×10^{-3}	0.0147	14.84	0.386
3#	Lime stone	2.31×10^{-4}	0.00953	11.51	0.31

*Experiments conducted by Gregorz (2002)
#Experiments conducted by Sarkar

was compared with the experimental data of the Dzido *et al.*[5] and also the earlier experimentation conducted by Sarkar[12]. Figure 1 shows that the estimated static pressure profile along the vertical riser as per the experimental data of Dzido *et al.*[5] which had shown similar trends but does not match exactly, particularly near the solid feeding point. It is observed that there is a marked difference along the small initial height starting from the solid feeding point, after which the deviation of same magnitude between experimental values and the values estimated by present model persists along the residual height. Such initial deviation may be explained by the fact that the present model assumes the particles to be uniformly distributed immediately after solid feed point. But during the actual experimentation, such ideal uniform distribution of particulate solids throughout the cross-riser section, immediately after introduction is very difficult. Rather, some portion of the cross-section is observed void of particles which decreases along the height and the particle assembly may appears almost like irregular inverted cone shaped and finally extending throughout the cross-section. Such non-uniformity sustains along this small height of riser near the feeding point, especially with higher solid loading. Thereby, the individual particles are not uniformly exposed to the up-flowing gas just after feeding point. As a result, the momentum force of the gas is not transferred to that extent to each individual particle for accelerating the solids, resulting in attainment of lower solid particle velocities than the expected values at any specific height as estimated by the model. This phenomenon of low transfer of momentum from the gas to particles in the actual experimentation is reflected in the lower values of static pressure drops in the actual experimentation compared to that proposed by the present model (contribution of the pressure drop due to particle acceleration in the total pressure drop is comparatively higher than other components in the initial section of riser as observed in Fig. 7). Such non-uniformity of the solid distribution near the feeding point may be more prominent in case of higher solid loading ratios. The computed static pressure profile in Fig.1 is characterized by nonlinearity with a gradual decreasing gradient in the initial section within 1meter. Further downstream, it becomes practically linear. But the experimental static pressure profile (PST) maintains the non-linearity with lower gradient till higher riser height signifying that the acceleration of the particles continues up to slightly higher riser height than that predicted by the model. Just to compare between the nature of the said experimental and theoretical profiles, the initial average difference between the computed and the aforesaid experimental values has been added in the model computation. Based on the adjusted or modified starting static pressure, new theoretical static pressure profile has been estimated to establish the probable justifications for the discrepancy. It is observed in Fig. 2 that the corrected estimated static pressure profile after this adjustment almost merges with the experimental static pressures values.

Validation of model with experimental data of Sarkar

The present theoretical model was also used to compare the similar experimental pressure drop values as conducted by Sarkar[12]. The theoretical pressure drop profiles estimated by the present model matches

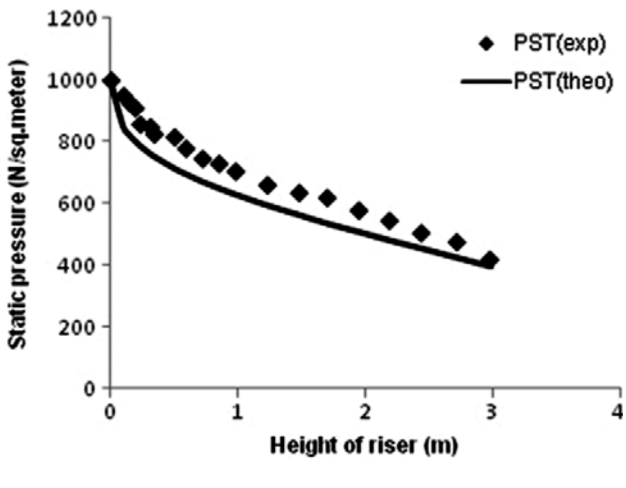

Fig. 1 — Comparison of experimental static pressure drop of Gregordz dzido *et al*. [2002] with estimated static pressure by present simulation.

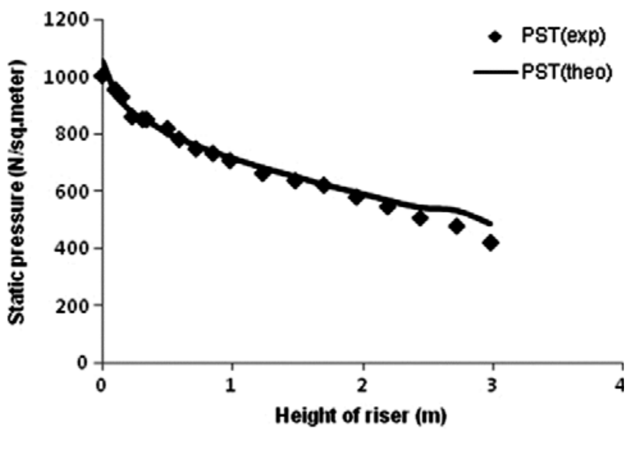

Fig. 2 — Merging of experimental points of Gregordzdzido *et al.* [2002] with estimated static pressure after incorporating necessary adjustment near the entry point.

comparatively better with the experimental pressure drop of Sarkar[12] with some minor deviations indicating comparatively lesser non-uniform particle distribution near the solid feeding point and the marked deviation in the earlier case (Figs. 1and 2) is not observed due to the fact that a special care was taken for better distribution of particles near solid feeding point (Fig. 3). Moreover, the solid loading SLR in such experiments are many times lower than the earlier experimentation of Gregorz Dzido and the particles are almost uniformly distributed near the solid feeding zone.

Analysis of the computational model

The simulation program based on the present model were run with various input data and the different computed hydrodynamic parameters like, total pressure drop, pressure drop components, particle velocity and particle Reynolds number profiles were analyzed.

The following observations and subsequent analysis were made on the graphical plots based on the data computed by the present theoretical model.

(i) Figure 4 shows the variation of total pressure drop along the height of the riser at different gas flow rates and at constant solid feed rate. In all the cases, the pressure drop gradually increases almost in the exponential fashion. With the increase of gas flow rate, the total pressure drop gradually increases with the height of riser. Initially, the rate of increase of pressure gradually decreases and after some initial height, it is constant. The probable reason for such a trend is that, up to a certain length, the particles get accelerated and the total pressure drop shows a gradient after which the particles attain steady state velocity and the pressure drop is almost proportional to the height of riser. This initial unsteady zone length is normally termed as 'acceleration length'.

(ii) Figure 5 depicts the variation of total pressure drop along the height of the riser at a constant gas flow rate but at varying solids feed rate. With the increase of solids feed, the total pressure at any height is higher in case of higher solids feed rate.

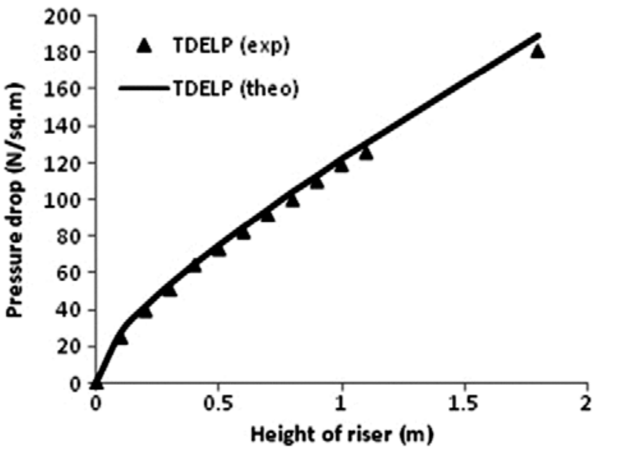

Fig. 3 — Comparison of experimental pressure drop of Sarkar [1994] with the data estimated by the present model.

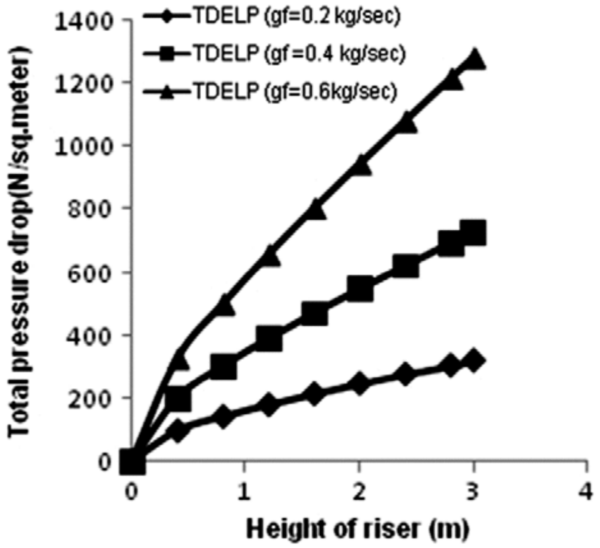

Fig. 4 — Variation of estimated total pressure drop with respect to riser height for different gas flow rate in the riser. [Fixed variables: d_p=1.61 mm, ρ_p=1240 kg/m^3,d_t= 2 inch]

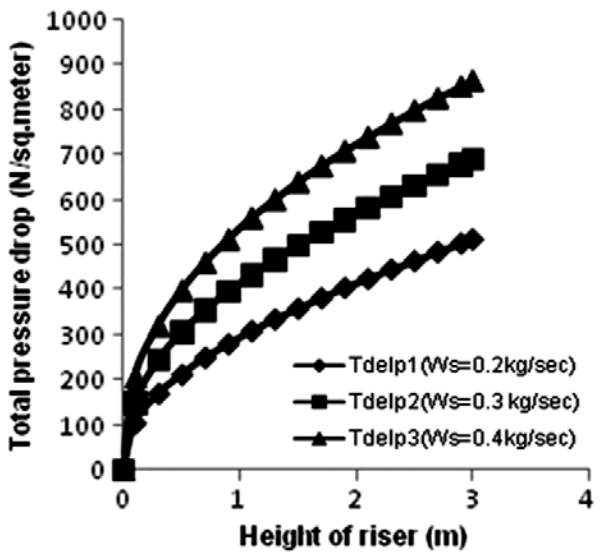

Fig. 5 — Variation of estimated total pressure drop with respect to riser height for different solids loading. [Fixed variables: d_p=1.61 mm, ρ_p=1240 kg/m^3,d_t=2 inch]

The lowest plot is with the minimum solids feed rate and the gradient persists up to a smaller initial length. But with the increase of solids feed rate, the gradient continues up to a higher height. This may be due to the fact that with the increase of solids loading, the particles are accelerated up to a higher height of the riser and gradient becomes constant beyond that height.

(iii) Figure 6 shows the variation of estimated particle velocity with riser height. With the increase of particle size, the particle velocity gradually reduces. The velocity profile also depicts that the higher the particle size, the lesser the particle velocity, and, the particle velocity gradient persists up to a slightly higher height.

(iv) Figure 7 describes the estimated pressure drops due to different components of pressure drops due to acceleration of particles, static pressure of gas, friction losses of the gas, friction losses of solids and the static pressure of the solids. The highest plot indicates that the total pressure drop is the summation of individual pressure drop components. The pressure drop component due to friction losses is almost proportional to the height of riser, which is shown by the plot DPFS. The pressure drop due to solid weight or static pressure is almost proportional to the height of the riser. In case of solids, the pressure drop due to weight of solids shows a slight gradient along the initial height of the riser. Beyond that it is almost proportional to the height of riser. The pressure drop due to acceleration of particles shows a trend similar to that of total pressure drop. The pressure drop due to particle acceleration has a major contribution in the total pressure drop. This plot shows the correct dynamic history of the particle which reflects that the particle gets accelerated up to a certain initial height beyond 0.5 m after which, the particle velocity attains an almost steady state and the gradient is almost constant throughout the rest of the height of riser.

(v) The estimated particle Reynolds number by the model has been plotted along the length of the riser in Fig. 8. The figure shows that along the initial height of the riser, the particle Reynolds number with respect to height is higher and shows a steep gradient up to a certain initial length

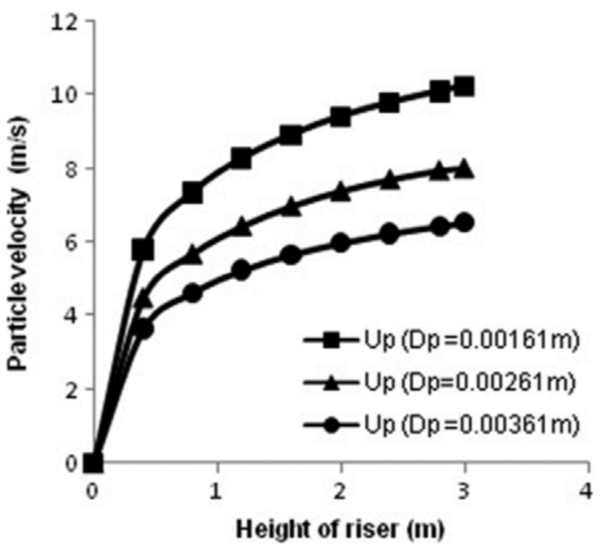

Fig. 6 — Variation of estimated particle velocity with riser height for different particle sizes in a vertical riser.

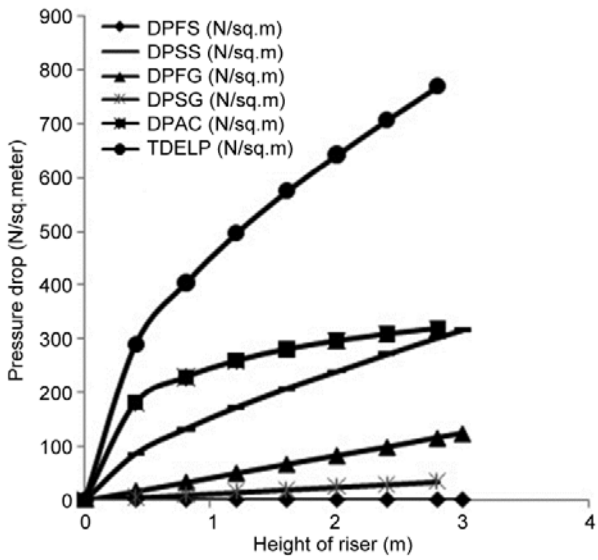

Fig. 7 — Variation of estimated total pressure and individual pressure components with riser height.

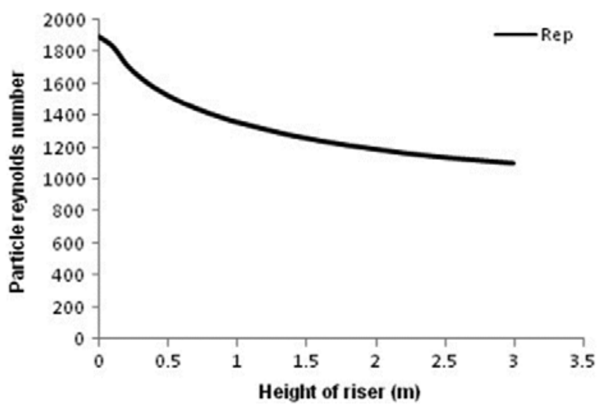

Fig. 8 — Variation of estimated particle Reynolds number (Re_p) with respect to riser height.

(accelerating height), after which, it increases slowly and tends towards an almost horizontal straight-line.

Conclusion

The computational algorithm, which has been developed for the present hydrodynamic model of vertical riser for dilute phase gas-solid flow, has been verified with the experimental results of other experimenters which show good agreement except some minor deviations at feed point in some cases. Some minor corrections at the particle entry point may be incorporated based on the actual experimental situations for exact fitting with its results. In addition, using the present theoretical approach, it is possible to analyze theoretically the various hydrodynamic parameters with possible justification successfully.

Nomenclature

d_p = particle Diameter(mm)
C_d = Drag Coefficient
C_{dm}= Modified drag co-efficient
d_t = Tube diameter(m)
ε = Void fraction
ρ_p= Particle density (kg/m^3)
f_s = Friction factor due to solids
f_g = Friction factor due to gas
ρ = Gas density (kg/m^3)
PST = Static pressure (N/m^2)
U_p = Particle velocity (m/s)
U_g = Superficial air velocity (m/s)
u_a = Actual air velocity through assembly of particles(m/s)
Re_p = Particle Reynolds number
Re_g = Gas Reynolds number
μ_g = Gas viscosity (pa.s)
Ws = Solids feed rate (kg/sec)
g_f = Air flow rate (kg/sec)
dp_{sg} = Differential pressure drop due to static column of fluid (N/sq.m)
dp_{fg}= Differential pressure drop due to the fluid friction with the wall (N/sq.m)
dp_{ss}= Differential pressure drop due to static column of solid particles (N/sq.m)
dp_{fs}= Differential pressure drop as a result of solid friction with the tube wall (N/sq.m)
dp_{as} = Differential pressure drop due to acceleration of the solid particles (N/sq.m)
dp_t = Total pressure drop across the element dh (N/sq.m)

Nomenclatures used in Figures

DPSG = Pressure drop due to static column of fluid (N/sq.m)
DPFG = Pressure drop due to fluid friction with the tube wall (N/sq.m)
DPSS = Pressure drop due to static column of solid particles (N/sq.m)
DPFS = Pressure drop as a result of solid friction with the tube wall (N/sq.m)
DPAC = Pressure drop due to acceleration of solid particles (N/sq.m)
TDELP = Total pressure drop (N/sq.m)

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