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Rheological model for studying fatigue of knitted fabric

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The plain knitted structure (cotton) generally utilized in the clothing domain has been studied with cyclic loading at diverse fatigue number cycles. Fatigue test induces delayed-elastic and permanent deformation that increase according to cyclic elongation. The delayed-elastic and permanent deformation of tested knit has been discussed based on Burger's rheological model. This model is suitable to describe the deformation behavior during relaxation of a plain knitted structure. The proposed rheological model permits us to estimate the elastic, viscoelastic and plastic components of the different tested fabrics after fatigue test.

Keywords: Burger's model, Delayed-elastic deformation, Fatigue test, Knitted fabric, Permanent deformation, Rheological model, Viscoelastic behavior

1 Introduction

The plain knitted fabrics made from cotton fibre are very popular in clothing industry. This fibre, having a non-thermoplastic property, is unable to be heat set. Dimensional properties of this type of knitted structure are extensively affected by applied stress that generates a permanent deformation even after a long period of relaxation. The knitted structures undergo many cyclic deformations during manufacturing or during body motion while wearing the clothes. A cyclic deformation of knitted structure can produce permanent deformation even after a long time of relaxation. This deformation that simulates the wearing of a knitted structure, can permit us to evaluate its viscoelastic behavior and its ability to recuperate its initial dimensions following cyclic loading.

To the best of our knowledge, literature does not report any process to evaluate the relaxation and the permanent deformation generated by repeated deformation that simulates the usage of knitted structure throughout the manufacturing process or during its use. Many studies examined the deformation of knitted structure at rest or under cyclic loading [1-4]. The relaxation properties of knitted structure have been studied by several researches based on the mechanical method. Hepworth & Leaf [5] and Postel & Munden .[6] considered in their proposed models that the filaments, composing the textile structure, show a perfect elastic deformation. In their study on the knitted fabrics, MarcRoy *et al*.[7] tried to model load-elongation properties of knitted structure. MacRoy's model was used to study slippage between knitting loops and applied stress on loop elements while deformation. To study the relaxation properties of knitted structure, Saber *et al*.[8] modeled the loop using mathematical functions. Considering that the deformation of a knitted fabric is viscoelastic [9,10], rheological modeling was based on the theory of viscoelasticity. The viscoelasticity is a necessary textile's physical property for real deformation behavior [11,12]. To model the viscoelastic properties of fabric, Vangheluwe & Kienkens.[13,14] used the generalized Maxwell model. This model does not consider the instantaneous and permanent deformation of textile structure during relaxation.

This paper reports study on the deformation during relaxation of knitted structure following cyclic loading, by using a fatigue tester and a rheological model. The used fatigue device allows us to achieve a very important number of cyclic loading. This procedure allows to evaluate the structure capacity to recuperate the original state following a cyclic

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loading, and to describe the deformation related to these cyclic tensile deformations and the components of plastic, viscoelastic and elastic deformation. This study will help clothing industry in presuming the knitted fabric properties.

2 Materials and Methods

2.1 Experimental Fatigue Device

The fatigue device (Fig. 1) was composed of a mobile and a fixed clamp. A mobile clamp is mounted to rotating disk via a carrier and a connecting rod. The two clamps were designed according to the French Norm NF G 07-140. Generally, these clamps are utilized to test a knitted structure using dynamometer. The motor, which generates the cyclic translation of a mobile clamps, runs at 100 rounds per minute. The displacement amplitude of the mobile clamp is variable between 40 mm and 80 mm. We tested the knitted structure with cyclic loading in longitudinal direction at five diverse number of cycles. The amplitude of the displacement of the mobile clamp is fixed to 60 mm. The tested sample had the following characteristics:

Yarn composition	: Cotton, 40 tex
Structure	: Plain knitted fabric
Gauge of machine	: 26 needles/inch
Row density	: 13.18/cm
Wale density	: 11.07/cm
Areal density	: 207g/m^2
Loop length	: 3.68 mm
Sample length	: 200 mm
Sample width	: 50 mm

Before each test, the sample of the size (50x50 mm) was used to study the dimensional variation during the fatigue test. To determine the rheological parameters, the fatigue test was followed by a creep test, performed with a constant force of 38N corresponding to 60 mm of elongation generated by the device during fatigue test. For the creep test, we utilize a dynamometer LRx2.5K (Lloyd, UK) at a speed of 100mm/s. Just after the fatigue test, each sample was laid out on flat and smooth surface to rest. During relaxation, we measured periodically the length of the drawn square along the longitudinal axis using a microscope (BaoMicT1-21TZ) associated with textile analysis software, without touching the sample. All the samples were then laid flat on the same smooth surface under the condition, having temperature 20±3° C and relative humidity 65±5%. Each value of all the parameters is the average of five measurements taken at the same relaxation time of samples that undergoes the same fatigue test (same number cycles).

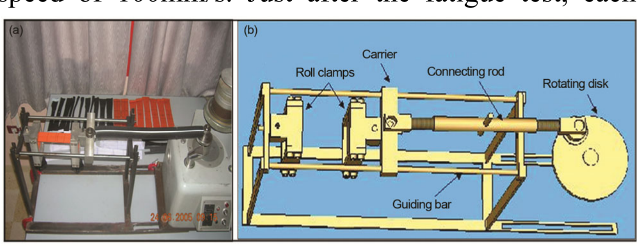

Fig. 1 —Fatigue device for fabric: (a) photography and (b) schematic diagram

2.2 Rheological Modelling

The deformation behavior of knitted fabric depends on its viscoelasticity characteristics [15,16]. In fact, by investigating the knitted structure recovery behavior during relaxation, it is found that the recovery was not immediate and was continued to change with the change in time. Therefore, this behavior can be modelled by spring (elastic component) and dashpot (viscous component) adequately connected to each other. In other studies, the textile fabric presents a nonlinear viscoelastic behavior [11,17,18]. There are several rheological models to study the mechanical properties of textile materials, such as Kelvin-Voigt's model, Maxwell's model and Burger's model. Based on the preliminary tests, it is found that the behavior of Burger's rheological model (Fig. 2) during recovery phase is similar to the deformation of plain knitted structure during relaxation.

In this study, the relaxation of knitted structure has been investigated along its ability to recover elongation and to attain to the initial state. Therefore, the rheological Burger's model was found to be the most suitable model to simulate relaxation behavior of the knitted structure. Therefore, we considered the rheological Burger's model to describe the viscoelastic properties of the knitted structure during relaxation.

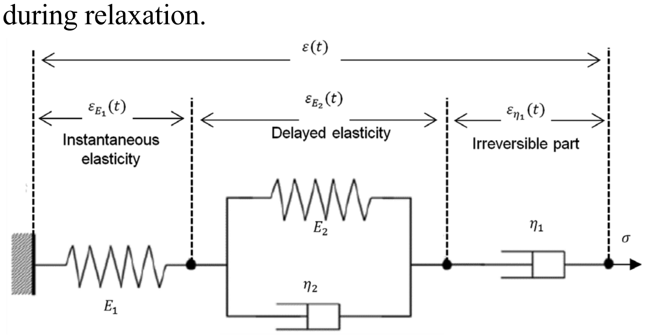

Fig. 2 — Burger's rheological model

Considering viscoelasticity properties of the textile structures, the rheological Burger's model, composed of Kelvin-Voigt model connected in series along with a Maxwell model, is studied in this research. The four rheological elements of Burger's model allowed us to investigate the viscoelastic behavior of knit's deformation during relaxation. This model includes Instantaneous elasticity characterized by the spring (E_1) and delayed elasticity (viscosity) characterized by the spring (E_2) and the dashpot (η_2), which are associated in parallel arrangement. The irreversible part (plasticity) is represented by the dashpot (η_1).

The combination of the dashpots and the springs in the rheological Burger's model was used to establish the following equations:

$$\sigma = \sigma_{\eta_1} = \sigma_{E_1} = \sigma_{\eta_2} + \sigma_{E_2} \quad \ldots (1)$$

$$\varepsilon = \varepsilon_{\eta_1} + \varepsilon_{E_2} + \varepsilon_{E_1} = \varepsilon_{\eta_1} + \varepsilon_{\eta_2} + \varepsilon_{E_1} \quad \ldots (2)$$

where E_2 and E_1 are the elasticity of springs (1 and 2); η_2 and η_1, the viscosity of dashpots (1 and 2); ε, the deformation; and σ, the stress.

$$\varepsilon_{E_1} = \sigma / E_1 \quad \ldots (3)$$

$$\varepsilon_{E_2} = \sigma_{E_2} / E_2 = (\sigma - \sigma_{\eta_2}) / E_2 \quad \ldots (4)$$

$$d\varepsilon_{\eta_1} / dt = \sigma / \eta_1 \quad \ldots (5)$$

Based on Eqs (1) -(5), we get:

$$\frac{d\varepsilon}{dt} = \frac{1}{\eta_1}\sigma + \frac{1}{E_1}\frac{d\sigma}{dt} + \frac{1}{E_2}\frac{d\sigma}{dt} - \frac{\eta_2}{E_2}\frac{d^2\varepsilon}{dt^2} + \frac{\eta_2}{E_2\eta_1}\frac{d\sigma}{dt} + \frac{\eta_2}{E_1E_2}\frac{d^2\sigma}{dt^2} \ldots (6)$$

Considering the above equations, Eq. (6) can be written in the following differential form, linking stress to deformation in rheological Burger's model:

$$\frac{d^2\varepsilon}{dt^2} + \frac{E_2}{\eta_2}\frac{d\varepsilon}{dt} = \frac{E_2}{\eta_1\eta_2}\sigma + \left(\frac{1}{\eta_1} + \frac{1}{\eta_2} + \frac{E_2}{E_1\eta_2}\right)\frac{d\sigma}{dt} + \frac{1}{E_1}\frac{d^2\sigma}{dt^2} \quad \ldots (7)$$

In a relaxation of knitted structure after cyclic loading, the deformation varies during the time. In order to characterize the relaxation during time using the elements of Burger's model, we formulated the deformation according to the constant stress σ_0 (the stress applied during creep test). The dependence of recovery deformation ε_{re} according to the time under the action of the constant stress σ_0 is provided by the following equation:

$$\varepsilon_{re}(t) = \frac{\sigma_0}{E_2}(1 - e^{-\frac{E_2}{\eta_2}t_1})e^{-\frac{E_2}{\eta_2}t} + \frac{\sigma_0}{\eta_1}t_1 \quad \ldots (8)$$

where t is the time during relaxation of knitted structure; and t_1, the duration of stress application during creep test.

In order to study the relaxation speed, we determined the slope (S_r) of relaxation evolution at the point (ε_{t_1}, t_1), using the following equation:

$$S_r = \frac{d\varepsilon_{re}}{dt}_{(\varepsilon_{t_1}, t_1)} = -\frac{\sigma_0}{\eta_2}(1 - e^{-\frac{E_2}{\eta_2}t_1})e^{-\frac{E_2}{\eta_2}t_1} \quad \ldots (9)$$

where ε_{t_1} is the deformation at a moment of the removal of stress (t_1).

3 Results and Discussion

3.1 Application and Validation of Rheological Model

Burger's model that simulates deformation of the knitted fabric during relaxation phase after cyclic loading, has been validated. The correlation has been studied among theoretical recovery curves, calculated from parameters of the Burger's model by using the Microcal Origin6 software and experimental relaxation curves. For this purpose, the correlation between the theoretical and experimental curves has been analyzed for two different samples which have undergone 6000 cycles of deformation. The parameters of rheological model have been calculated as of experimental data. One of the samples undergoes cyclic deformations of amplitude 40 mm and the other sample undergoes cyclic deformations of amplitude 60 mm. Figure 3 shows the evolution of the experimental and fitted relaxation according to the relaxation time of two different tests.

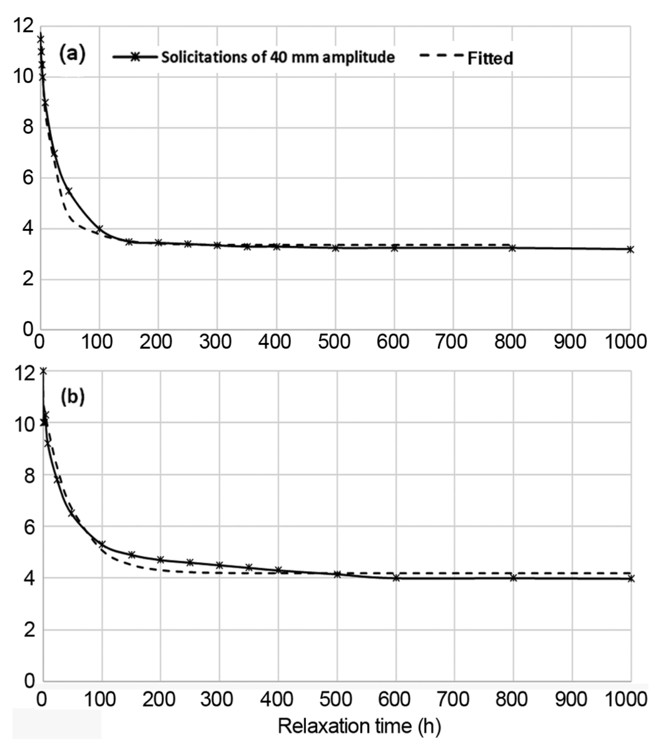

Fig. 3 — Non-linear fitting relaxation curve of knitted fabric following 6000 fatigues cycles (a) cyclic deformations of amplitude 40 mm and (b) cyclic deformations of amplitude 60 mm.

The coefficient of correlation (R^2) has been used to compare the experimental elongation with that fitted of two tested samples. Fitting aptness can be assessed by comparing the coefficient of correlation (R^2). The higher the coefficient (≈1), the better the fit is obtained. The calculated values of the model parameters and the values of R^2, during relaxation following 6000 cycles deformation for two amplitudes elongation, are summarized in Table 1. The Burger's model appears to be a great tool for studying the relaxation after fatigue test of knitted structure. In fact, for the two curves in figure 3, R^2 exceeds 97%. Therefore, this model has been used to evaluate the effect of number of deformations on viscoelastic knitted properties and its relaxation after the removal of stress.

3.2 Burger's Model to Study Relaxation Knitted Fabric following Fatigue Test

The knitted fabric is tested at five different cycles for amplitude of 60 mm. The rheological Burger's model is used to study the effect of cyclic deformation on relaxation behavior. Experimental relaxation curves are used to identify the viscoelastic parameters. Using dependence of recovery deformation equation [Eq. (8)] and Microcal Origin6 software, the values of E_2, η_2 and η_1 are calculated for different number of fatigue cycles. Figure 4 shows the calculated and experimental relaxation curves for different fatigue number cycles. Repeated traction shows a notable influence of the knitted structure to recover its primary length. Indeed, fatigue test generates elongation in longitudinal direction and contraction in transverse direction of the sample. Part of generated deformation is conserved after the removal of cyclic deformations, but decreased gradually with the time; it's the phenomenon of relaxation. The deformation, during relaxation, decreases quickly between 0 h and 500 h. Then, it decreases gradually but never returns to its initial state and never reaches value relaxation with the different fatigue test number of cycles.

Table 1 — Parameters of Burger's model applied to knitted fabric following 6000 cycles deformations at various amplitudes

Rheological parameters	40 mm amplitude	60 mm amplitude
E_2 , N.m^{-2}	0.105	0.114
η_2 , N.s.m^{-2}	3.365	5.680
η_1 , N.s.m^{-2}	8.056	9.070
R^2	97%	97%

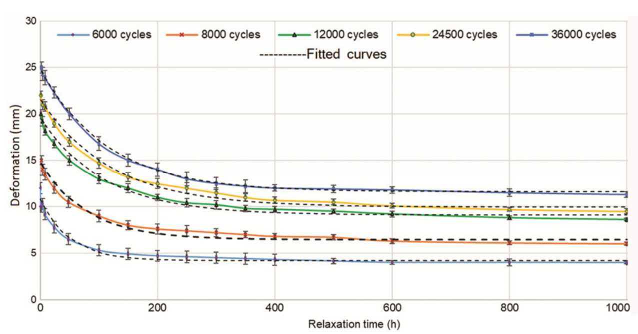

Fig. 4 — Evolution of fitted and calculated curves relaxation of knitted fabric after fatigue test

Considering the knitted structure that presents a viscoelastic property, the recovery of the deformation, following fatigue test, is not immediate (Fig. 5). It decreases with the time (delayed-elastic deformation) till stabilization at a new state, which is different from the initial state (permanent deformation).

The value of delayed-elastic deformation corresponds to the value of deformation after a long period of relaxation ($t_{\rightarrow+\infty}$). In our case, the time reaches to infinite ($t_{\rightarrow+\infty}$) corresponds to the last recovery value ($t \approx 1000\,h$), when the deformation stabilizes and it takes an asymptotic shape. If the time reaches to infinite, the delayed-elastic deformation is recovered entirely and the deformation ε_{re} tends to $\frac{\sigma_0}{\eta_1}t_1$ [Eq.(8)]. This calculated deformation increases according to the number of fatigue cycles. The experimental data of permanent deformation is in accordance with its calculated values [Fig. 6 (b)].

In fact, the rheological parameters and the tangent to the curve at the beginning of relaxation (ε_{t_1}, t_1) [Eq.(9)] decrease according to cyclic deformations (Fig. 7).

The estimated values of rheological model components, the slope at relaxation curve S_r and the values R^2 are summarized in Table 2. Rheological Burger's model appears a great choice for studying relaxation behavior following fatigue test of knitted fabric. Indeed, the value R^2 always exceeds 97%.

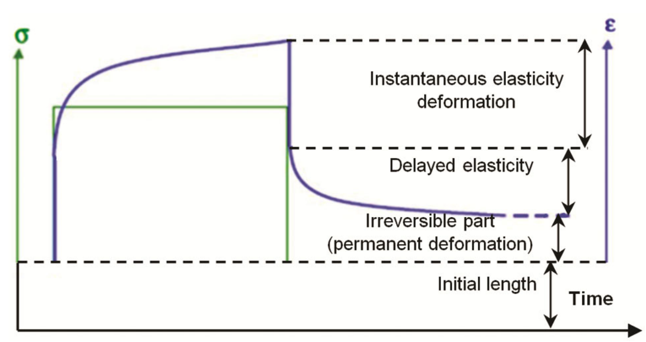

Fig. 5 — Burger's model deformations during a creep-recovery test.

Figure 7 shows the evolution of elastic component (E_2), the viscous components (η_1 , η_2) and the slope of the relaxation curve (S_r) according to cyclic deformations. The deformation generates a new viscoelastic behavior of knitted structure. In fact, elastic component, viscous components and slope of the relaxation curve decrease according to the number of cycles. The four factors decrease rapidly between 6000 cycles and 12000 cycles. After 12000 cycles, the decrease is found slow.

After deformation, knitted structure does not immediately recover to its original state (Fig. 4). On removing the loading stress, there are three phases of recovery. An instantaneous recovery which is

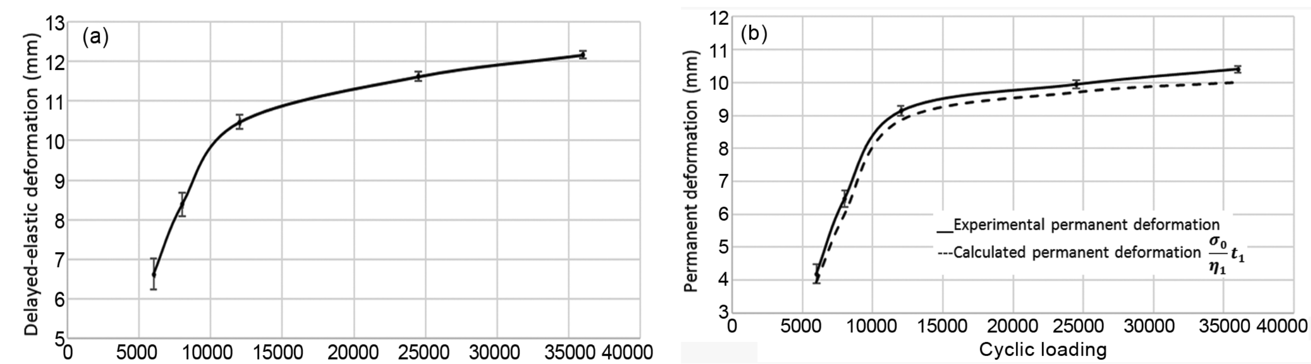

Fig. 6 — Evolution of delayed-elastic (a) and permanent deformation (b) according to the number of fatigue cycles

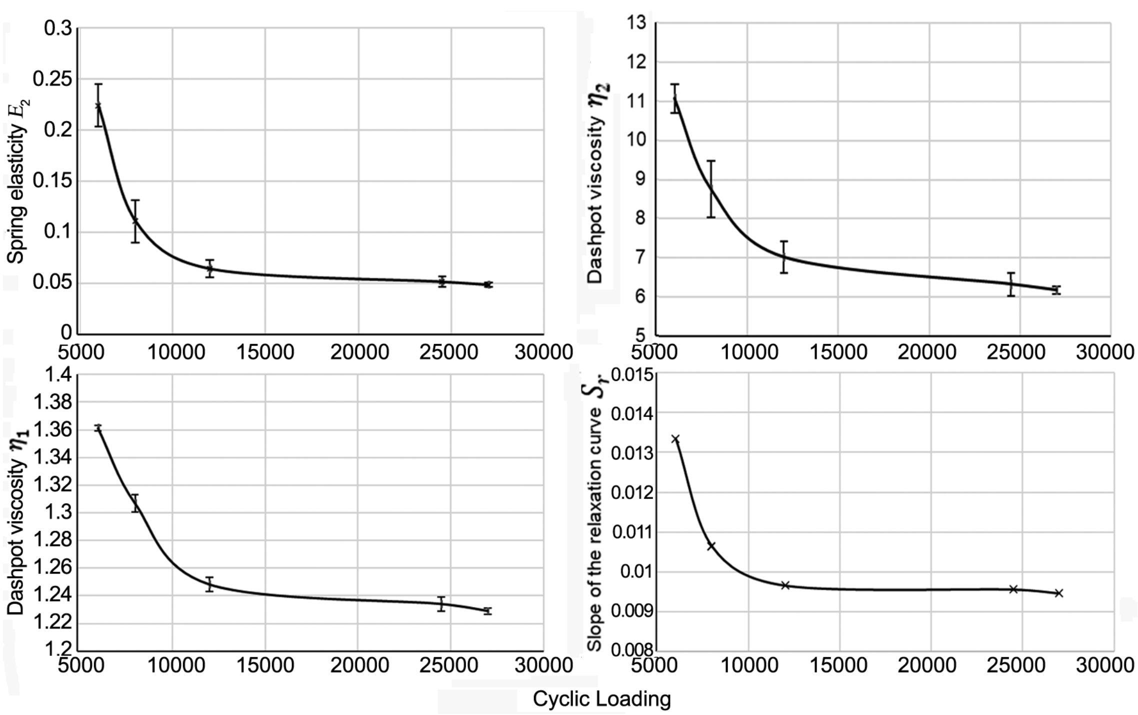

Fig. 7 — Evolution of Burger's model parameters according to number of fatigue cycles

Table 2 — Burger's model parameters for different number of fatigue cycles

Number fatigue cycles	Rheological parameters				R^2 %
	E_2, N.m^{-2}	η_2, N.s.m^{-2}	η_1, N.s.m^{-2}	$absolute(S_r)$, mm.s^{-1}	
6000	0.1135	5.68	9.07	0.1297	97
8000	0.0572	4.5	5.87	0.1053	97
12000	0.0333	3.613	4.16	0.0956	97
24500	0.027	3.256	3.82	0.0956	98
36000	0.024	3.11	3.65	0.0932	99

modeled by the elastic component (E_1). A viscoelastic recovery (delayed-elastic deformation), modeled by a spring (E_2) and a dashpot (η_2), evolves with a logarithmic shape according to the time relaxation. Unrecovered deformation (permanent deformation), which represents the viscous component of the structure, is modeled by the dashpot (η_1).

The partial recovery, during relaxation, of the knitted structure after the fatigue test can be explicated by the viscous and the viscoelastic characteristics of knit. There are two components of viscoelastic properties: (i) the viscosity (η_2) of the dashpot which connected in parallel with the spring (E_2), which represents the elastic behavior; (ii) The dashpot (η_2) which evolves according to the evolution of the spring (E_2) , and explains the viscoelastic behavior of the structure. The deformation of the knit fabric changes rapidly at the begin of relaxation and then takes an asymptotic appearance but never reaches the initial state. The relaxation rate, just a moment after the removal of stress, depends on the two viscoelastic components, namely (E_2) and (η_2) [Eq.(9)]. When the viscoelastic components decrease according to the number of fatigue cycles, the speed of relaxation decreases in the same manner. Therefore, the slope at the relaxation curve (S_r) decreases as a function of cyclic deformations (Fig. 7).

The permanent deformation can be explained by hysteresis phenomenon. This hysteresis can be explained by the friction between fibres in the yarns during cyclic deformations. In the recent studies, Morton and Hearle [19] show that the cyclic loading produces inter filaments friction, which increases the internal temperature of the fabric. This temperature enhances the phase shift of stress-deformation and opposes structure to attain its initial state.

Using the parameters of Burger's model (Table 2) and the recovery equation [Eq. (8)], the predicted values of delayed-elastic and permanent deformation can be estimated during relaxation. Therefore, delayed-elastic and permanent deformation can be predicted using recovery equation of Burger's model. Note that after the removal of stress, the sample has a delayed-elastic deformation which decreases over the time. In addition, the deformation recovery is not complete even after a long relaxation time; it's the permanent deformation. Indeed, after 1000 hours of relaxation the tested fabric does not return to its initial state. This behavior indicates the imperfect elasticity of textile structures.

4 Conclusion

A fatigue device that simulates the knitted structure deformation during manufacturing and throughout clothing use has been studied. The fatigue test is related to Burger's rheological model in order to study a viscoelastic behavior of the knit structures following cyclic deformations. The study reported in this paper is a practical tool that can be used by garment manufacturers to estimate long-term recovery of the knitted structure used in stretchable clothes, such as sportswear and underwear. The cyclic loading applied to a knitted structure involves a change in fabric dimensions, which manifests the change in delayed-elastic deformation curves. This deformation decreases according to the time till a permanent deformation is achieved after a long period of relaxation. The permanent deformation depends intensely on the viscous properties and the number of repetitive loading.

A nonlinear rheological model has been proposed with the aim to study the recovery under relaxation of textile knitted fabric. The Burger's model is a good choice for simulating the deformation according to the time during relaxation. Founded on the parameters of rheological model identified from experimental studies and the recovery equation, the estimated values of delayed-elastic and permanent deformation are calculated during relaxation. This model permits us to calculate the viscoelastic, plastic and elastic components for each tested structure. Besides, the greater the number of cyclic deformations, the more is the delayed-elastic and permanent deformation, during relaxation, increase. For modeling of fibres slippage in the yarns and fibres, viscoelasticity is needed to know viscoelastic and hysteresis phenomena of yarns and fibres to estimate knitted structure dimensional behavior from yarn and fibre properties.

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