Indian Journal of Pure & Applied Physics
Vol. 60, May 2022, pp. 449-454

Efficiency and Irreversibility Analysis of MHD Unsteady Fluid Flow with Heat and Mass Transfer

Madhurya Hazarika* & Debasish Dey
Department of Mathematics, Dibrugarh University, Dibrugarh, Assam- 786 004, India

Received 27 October 2021; accepted 19 April 2022

In this paper the analysis of entropy generation of unsteady MHD fluid flow passing through the stretching sheet under the influence of Soret and the chemical reaction has been investigated. The multiple slip effects, suction/injection are considered to analyze the entropy generation rate. The non-linear governing equations are transformed into the first order differential equations using suitable similarity transformations. The resulting boundary value problem is solved by using the MATLAB built in bvp4c solver technique. Numerical values of skin friction coefficient, Nusselt number, Sherwood number and irreversibility ratios are shown in tabular form. Further uniqueness of this work is to find the various irreversibility ratios at the surface due to the effect of thermal and mass diffusion, magnetic field, combined effect of heat and mass transfer over total entropy generation number. It is noticed that to reduce the irreversibility and to boast up the efficiency we have to control the values of chemical reaction parameter (K_1), Prandtl number and suction/injection parameter. Irreversibility due to combined effect of heat and mass transfer contributes more wastage than others.

Keywords: Entropy generation; Efficiency; Heat and mass transfer; Multiple slip effects; bvp4c

1 Introduction

Entropy generation analysis method is employed to reduce the wastage production of the engineering devices. The optimal performance of thermodynamics devices is obtained by reducing the rate of entropy generation. Bejan[1,2] was the first who introduce the idea of minimization of entropy generation. In recent years various researchers have been inspired to perform research on different applications of generation of entropy. Srinavasacharya & Bindu[3,4] have studied the second law analysis of micropolar fluid flow. Rashidi[5], Govindaraju *et al.*[6,7], Afridi & Qasim[8] & Abolbashri *et al.*[9] have examined the production of entropy in the presence of both Newtonian and non-Newtonian fluid flow model. Recently Dey & Hazarika[10] have analyzed the entropy generation of micropolar fluid flow using bvp4c solver. This method is well established procedure in thermal systems and engineering. Entropy generation analysis plays an important role in thermal machines such as power plant, refrigerators, heat pumps, air conditioners etc. The main sources of irreversibilities are due to (i) heat transfer, (ii) imposition of magnetic field, (iii) fluid friction irreversibility, (iv) diffusive irreversibility *etc*. The efficiency of energy transportation in a system can be improved by minimization of wastage production.

Fluid flow with multiple slip effects has several applications in engineering and industrial systems. Navier[11] introduced the concept of no slip condition to incorporate the possibility of fluid flow in the solid boundary. The slip can occur if the working fluid contain concentrated suspensions. Slip effects of MHD fluid flow over a flat plate has been studied by Das *et al.*[12]. Zhu *et al.*[13] & Chauhan & Olkha[14] have examined the simultaneous effect of partial slip on fluid flows.

Nowadays the applications in science and engineering the MHD fluid flow over a stretching surface have attracted many researchers. Stretching sheets have been widely used in the process of the boundary layers through liquid firm, concentration process, aerodynamics extructions of plastic sheets etc. Many researchers have shared their ideas and important findings of MHD fluid flow problems by considering the stretching sheets with heat and mass transfer. Some significant researches on fluid flows over a stretching sheet are reported in Refs.[15–18]. In this paper we have extended the works of Mabood & Sateyi[19] by analyzing the irreversibility with heat and mass transfer under the effect of multiple slip, soret

*Corresponding author:
(E-mail: madhuryahazarika1995@gmail.com)

and chemical reaction parameter with the help of MATLAB built in bvp4c solver technique. A comparison is presented with the previous work done by Mabood & Shateyi[19] & Ali[20] we have obtained a good agreement with the previous result.

2 Formulation of the problem

A two-dimensional MHD flow of an incompressible electrically conducting fluid over a permeable stretching surface in the occurrence of chemical reaction has been considered. The x-axis is measured along the sheet, and y-axis is normal to it as shown in Fig. 1. Sheet is moving with the velocity $U(x) = \frac{ax}{1-ct}$. A Transverse magnetic field $B(x) = B_0 x^{-\frac{1}{2}}, (x \neq 0)$ has been applied, where $B_0 \neq 0$ .Let us assume that the induced magnetic field is negligible as compared to the applied magnetic field. The governing equations for the continuity, momentum, energy, and species concentration are as follows (Maboob & Shateyi[19]):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad ...(1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial y^2}\right) - \sigma B_0^2 \frac{u}{\rho} + g\beta_T (T - T_\infty) + g\beta_c(C - C_\infty) \qquad ...(2)$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 - \sigma B_0^2 \frac{u^2}{\rho} \qquad ...(3)$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} - k_1(C - C_\infty) \qquad ...(4)$$

The relevant Boundary Conditions are

At $y = 0: u = U_w + u_{slip}, v = v_w, T = T_w + T_{slip}, C = C_w + C_{slip}$ ...(5)

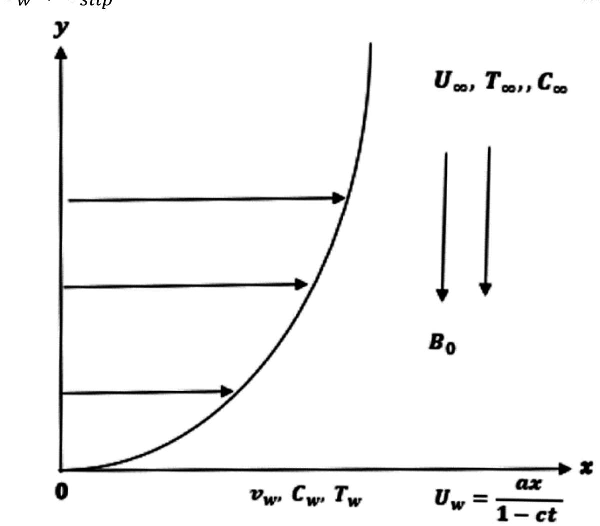

Fig. 1 — Flow diagram.

At $u \to 0, T \to T_\infty, C \to C_\infty$ ...(6)

Using the similarity transformations,

$$T_w = T_\infty + T_0\frac{ax}{2\nu\,(1-ct)^2}, C_w = C_\infty + C_0\frac{ax}{2\nu\,(1-ct)^2}$$

$$\eta = y\sqrt{\frac{a}{\nu_f(1-ct)}}, \psi = \sqrt{\frac{a\nu_f}{1-ct}}\,xf(\eta), \theta\,(\eta) = \frac{T-T_\infty}{T_w-T_\infty}, \phi = \frac{C-C_\infty}{C_w-C_\infty}, \qquad ...(7)$$

The non-linear partial differential equations (1)-(4) with the relevant boundary conditions can be transformed into the set of ordinary differential equations:

$$f''' = (f'^2 - ff'') + \lambda\left(f' + \frac{\eta}{2}f''\right) - Gr\theta - Gm\,\phi - Mf' \qquad ...(8)$$

$$\theta'' = \Pr\left[f'\theta - f\theta' + \lambda\left(\frac{\eta}{2}\theta' + 2\theta\right) - Mf'^2 - Ecf''^2\right] \qquad ...(9)$$

$$\phi'' = Sc\left[(f'\phi - f\phi') + \lambda\left(\frac{\eta}{2}\phi' + 2\phi\right) - S_0\theta'' + K_1\phi\right] \qquad ...(10)$$

Boundary conditions:

$$f(0) = f_w, f'(o) = 1 + ef''(0), \theta(0) = 1 + e_1\theta'(0), \phi(0) = 1 + e_2\phi'(0) \qquad ...(11)$$

$$f' \to 0, \theta \to 0, \phi \to 0 \text{ at } \eta \to \infty \qquad ...(12)$$

where, $Pr = \frac{\nu}{\alpha}, Gr = \frac{g\beta_T T_0}{a\nu}, Gm = \frac{g\beta_c C_0}{a\nu}, \lambda = \frac{c}{a}, Sc = \frac{\nu}{D_m}, S_0 = \frac{D_T T_0}{\nu C_0}, M = \frac{\sigma B_0^2(1-ct)}{\rho a}$

The volumetric entropy generation rate per unit volume can be defined as (Bejan[1,21], Dey & Hazarika[10]

$$S_G = \frac{k_f}{T_\infty^2}\left(\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right) + \frac{\mu}{T_\infty}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{(\sigma B_0^2 u^2)}{T_\infty} + \left(\frac{RD}{T_\infty}\left(\frac{\partial T}{\partial x}\frac{\partial C}{\partial x} + \frac{\partial T}{\partial y}\frac{\partial C}{\partial y}\right) + \frac{RD}{C_\infty}\left(\frac{\partial C}{\partial x}\right)^2 + \frac{RD}{C_\infty}\left(\frac{\partial C}{\partial y}\right)^2\right) \qquad ...(13)$$

Where the first, second, third and fourth terms of the equation (13) represent irreversibility due to heat transfer, energy dissipation, imposition of magnetic field and combined effect of heat and mass transfer.

Using the similarity transformation, we have the entropy generation number.

$$Ns = \frac{S_G}{S_{G_0}} = \left(\frac{1}{X}\right)^2 [\left(\theta(\eta)\right)^2 + Re\left(\theta'(\eta)\right)^2] + \left(\frac{Br}{\Omega}\right)(f'')^2 + \frac{M\,Br}{\Omega}f'^2 + \left(\frac{1}{X}\right)^2\left(\frac{\chi}{\Omega}\right)\lambda_1\left[(\theta(\eta)\phi(\eta) + Re\,\theta'\phi') + \left(\frac{\chi}{\Omega}\right)(\phi^2 + Re\,\phi'^2)\right] \qquad ...(14)$$

Where $S_{G_0} = k_f\frac{(T_w-T_\infty)^2}{L^2 T_\infty^2}$, $X = \frac{x}{L}, Re = \frac{U_w x}{\nu}, Br = \mu\frac{U_w^2}{k_f(T_w-T_\infty)}, \Omega = \frac{\Delta T}{T_\infty}, \chi = \frac{\Delta C}{C_\infty}, \lambda_1 = RD\frac{C_\infty}{k_f}$.

Bejan number (Be) is the ratio of irreversibility due to heat transfer to the total entropy generation number (Ns)

$$Be = \left(\frac{1}{X}\right)^2 \frac{\left[\theta(\eta)^2 + Re\,\left(\theta'(\eta)\right)^2\right]}{Ns}$$

Here we have considered the irreversibility ratios due to energy dissipation (Be_1), magnetic field (Be_2) and combined effect of heat and mass transfer Be_3.

$$Be_1 = \frac{\left(\frac{Br}{\Omega}\right)(f'')^2}{Ns}$$

$$Be_2 = \frac{\frac{MBr}{\Omega} f'^2}{Ns}$$

$$Be_3 = \frac{\left(\frac{1}{X}\right)^2 \left(\frac{\chi}{\Omega}\right)\lambda_1 \begin{bmatrix} (\theta(\eta)\phi(\eta) + Re\,\theta'\phi') + \\ \left(\frac{\chi}{\Omega}\right)(\phi^2 + Re\,\phi'^2) \end{bmatrix}}{Ns}$$

The local skin friction co-efficient (C_f), Nusslet Number (Nu) and Sherwood number are defined as follows:

$$C_f = \frac{\mu}{\rho\, U_w^2}\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$Nu_x = \frac{x}{k_f(T_w - T_\infty)}\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

$$Sh_x = -\frac{x}{C_w - C_\infty}\left(\frac{\partial C}{\partial y}\right)_{y=0}$$

Now using the similarity transformations, the reduced skin friction coefficient (C_{rfx}), Reduced Nusselt Number (Nu_{rx}) and Sherwood number (Sh_{rx}) are defined as

$Nu_{rfx} = f''(0), Nu_{rx} = \frac{Nu_x}{\sqrt{Re_x}} = -\theta'(0)$ and $Sh_{rx} = \frac{Sh_x}{\sqrt{Re_x}} = -\phi'(0)$

3. Numerical Methodology

In this paper we have used the MATLAB built in bvp4c solver technique to solve the problem. First of all, we have expressed the transformed system of ordinary differential equation in terms of first order ordinary differential equation .(Shampine & Keirzenka[22], Dey *et al.*[23] & Dey & Chutia[24]

$$f = z_1, f' = z_1' = z_2, f'' = z_2' = z_3, \theta = z_4, \theta' = z_4' = z_5, \phi = z_6, \phi' = z_6' = z_7$$

$$f''' = z_3' = (z_2^2 - z_1\,z_3) + \lambda\left(z_2 + \frac{\eta}{2}z_3\right) - Gr\,z_4 - Gm\,z_5 - M\,z_2$$

In similar way we can express the transformed equations (9), (10) in terms of first order ordinary differential equations. The boundary conditions can be expressed as

$$z_1(0) = f_w, z_2(0) = 1 + e\,z_3(0)$$
$$z_2(\infty) \to 0$$

4 Results and Discussion

The non- linear differential equations along with the relevant boundary conditions are solved using MATLAB built in bvp4c solver technique. Table 1 depicts the numerical values of Nusselt number ($-\theta'(0)$). To validate the results, we have considered: $e = e_1 = e_2 = f_w = \lambda = M = k_1 = Ec = \lambda_1 = \lambda_2 = 0$. Numerical results show a better agreement with the previous work done by Mabood & Shateyi[14] & Ali[15].

The fixed values assign to the physical parameters are $M = 2, Sc = 0.60, \lambda = 0.5, S_0 = 0.6, Gr = 0.5, Gm = 0.6, Ec = 0.2, Pr = 0.71, Br = 2, Re = 2, X = 0.5, e = 0.5, e_1 = 0.6, e_2 = 0.5, f_w = 1.5$. Here, we have calculated the values of different irreversibility ratios at the surface of the stretching sheet. In Table 2 we have presented the values of skin friction co-efficient, Nusselt number and Sherwood number for various values of M, Pr, Ec and Sc. It is seen that the rate of heat transfer enhances for the rising values of Pr and Sc but in case of Magnetic parameter (M) and Eckert number (Ec) an opposite result is obtained. Schmidt

Table 1 — Comparison the values of Nusselt number ($-\theta'(0)$) for various values of Prandtl Number (Pr) when $e = e_1 = e_2 = f_w = \lambda = M = k_1 = Ec = \lambda_1 = \lambda_2 = 0$

Pr	Ali[20]	Mabood & Shateyi[19]	Present Result
0.71	0.8058	0.8088	0.8048
1.0	0.9691	1.0000	1.0005
3.0	1.9144	1.9237	1.9235
10	3.7006	3.7207	3.7205

Table 2 — Numerical values of Skin friction, Nusselt Number and Sherwood number at the surface ($\eta = 0$) for different values of M, Pr, Ec and Sc

M	Pr	Ec	Sc	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
1				-1.3037	0.8531	0.6500
4				-1.6727	0.7621	0.6340
6				-1.8400	0.7299	0.6263
	0.71			-1.4498	0.8134	0.6460
	1			-1.4547	0.8980	0.6154
	3			-1.4587	1.1687	0.4651
		0.1		-1.4514	0.8285	0.6477
		0.2		-1.4498	0.8134	0.6460
		0.3		-1.4481	0.7984	0.6443
			0.60	-1.4498	0.8134	0.6460
			1.0	-1.4672	0.8142	0.7396
			1.5	-1.4804	0.8148	0.8189

number plays positive impact on rate of heat transfer is greatly affected by the increasing values of Sc but magnetic parameter, Prandtl number and Eckert number influences to diminish the rate of mass transfer. We also observe that, with the enhancement of the magnetic parameter M, Prandtl number Pr and Schmidt number Sc, the skin friction significantly increases.

Table 3 portrays the irreversibility ratios at the surface of the stretching sheet ($\eta = 0$) for different values of Magnetic parameter M, multiple slip parameters for velocity, temperature and suction/injection parameter f_w. For all the parameters M, f_w and e mentioned in the table 3, Bejan number shows enhancement but a negative impact is found in case of temperature slip parameter (e_1) which will boost the efficiency of the system. Suction injection parameter f_w, temperature slip $e_1$and magnetic field parameter(M) shows positive impact on the values of Be_1 but in case of velocity slip parameter e, this value experiences decreasing trend. Imposition of magnetic field results the reduction of irreversibility and enhancement of the efficiency of the system. Irreversibility due to combined effect of heat and mass transfer (Be_3) contributes more wastage production than others which will reduce efficiency of the system.

Figures 2-7 demonstrate the entropy generation number against η for various values of Magnetic parameter M, chemical reaction parameter K_1, Prandtl number Pr, velocity slip parameter e, temperature slip parameter e_1, suction/injection parameter f_w and concentration slip parameter e_2. Fig. 2 depicts that the Ns is decreasing near the surface of the stretching sheet for rising values of M. This can be explained

Table 3 — Numerical values of Bejan number (Be), Be_1, Be_2 and Be_3 at the surface of the stretching sheet ($\eta = 0$).

M	f_w	e	e_1	Be	Be_1	Be_2	Be_3
1				0.1526	0.0735	0.4060	0.3679
2				0.1558	0.0948	0.3640	0.3855
3				0.1585	0.1130	0.3381	0.4096
	0.5			0.1770	0.1147	0.1513	0.5570
	1.0			0.2018	0.1268	0.1187	0.5526
	1.5			0.2258	0.1361	0.0899	0.4762
		0.1		0.2056	0.1555	0.0640	0.5750
		0.3		0.2448	0.0819	0.0410	0.6320
		0.5		0.2629	0.0502	0.0298	0.6715
			0.2	0.2685	0.0408	0.0261	0.6645
			0.4	0.1964	0.0474	0.0284	0.7277
			0.6	0.1490	0.0522	0.0299	0.7087

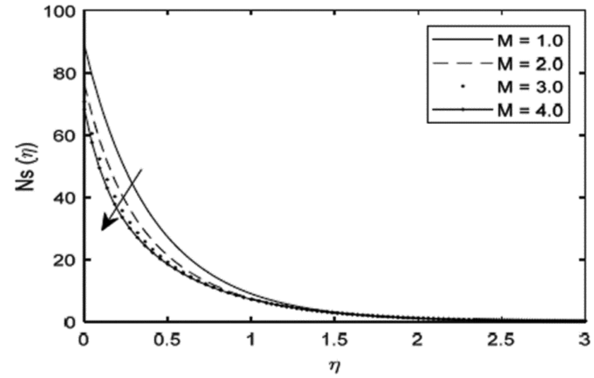

Fig. 2 — Entropy generation against η for different M.

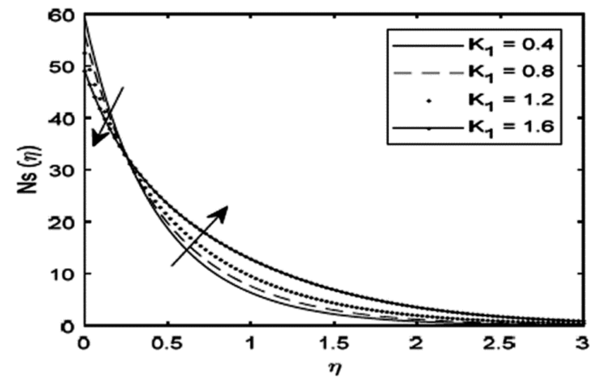

Fig. 3 — Entropy generation against η for different K_1.

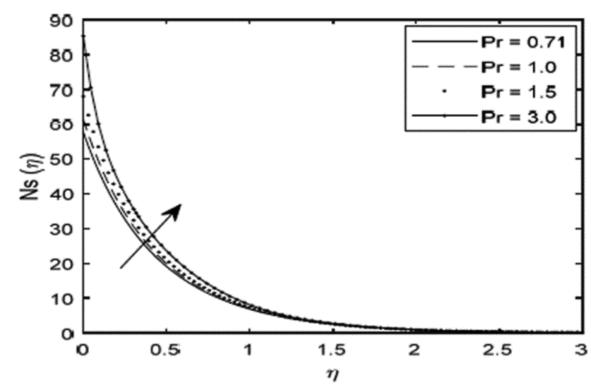

Fig. 4 — Entropy generation against η for different Pr.

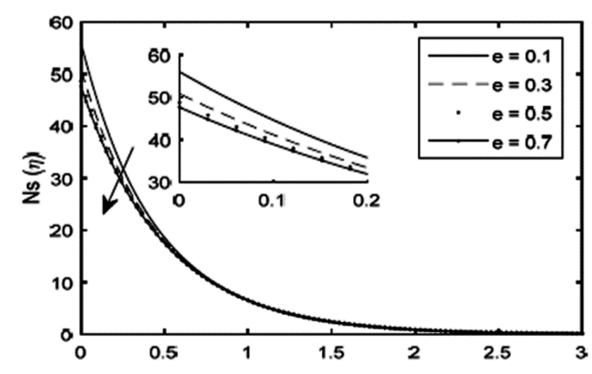

Fig. 5 — Entropy generation against η for different e (Velocity slip).

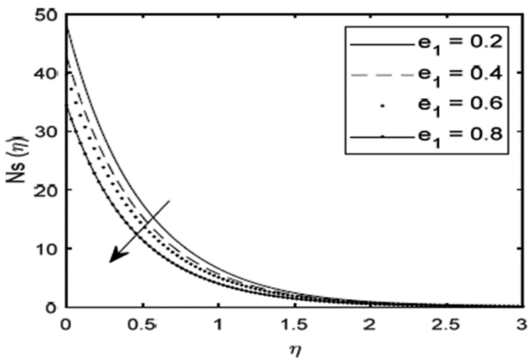

Fig. 6 — Entropy generation against η for different e_2 (Temperature slip).

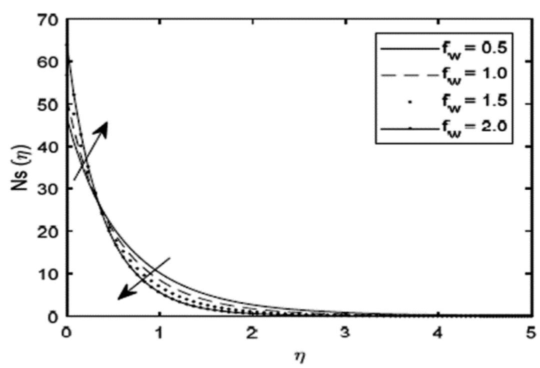

Fig. 7 — Entropy generation against η for different f_w (Suction/injection).

from the physics of the problem, since by applying the transverse magnetic fields leads to form the Lorentz force which tends to resist the fluid flow and temperature increases. Consequently, the resulting entropy generation rate decreases. Fig. 3 reveals that rate of entropy generation follows reduction near the surface of the stretching sheet with the increment of chemical reaction. Fig. 4 shows that Prandtl number (Pr) helps to enhance the entropy generation and reduce the efficiency. Figs 5 & 6 depict the effect of slip parameters for velocity and temperature on entropy generation number. It is noticed from the figures that both the slip parameters inversely effect on entropy generation and directly proportional to the efficiency.

Maximum value of entropy generation number is observed in the vicinity of the stretching sheet for rising values of f_w. Furthermore, moving away from the surface to the edge of the boundary layer entropy generation number shows reduction (Fig. 7).

5 Conclusions

The following conclusions are obtained based on the graphical tabular results:

1. Shear stress decreases as Ec enhances and to reduce the shear stress we have to control the values of M, Pr and Sc.
2. The rate of heat transfer at the surface enhances as a result of increasing Pr and Sc and decreases for M and Ec.
3. Irreversibility due to combined effect of heat and mass transfer contributes more wastage production than others.
4. To reduce the irreversibility and to boast up the efficiency we have to control the values of chemical reaction parameter (K_1), Prandtl number and suction/injection parameter (f_w).
5. Entropy generation number reduces for the increasing values of Magnetic parameter M, multiple slip parameters for velocity and temperature.

Nomenclature

c Stretching rate
B_0 Magnetic field of constant co-efficient
Br Brinkman number

Greek Symbols

η Similarity variable
μ viscosity
ν kinematic viscosity
α Thermal diffusivity
β_T Thermal expansion co-efficient
β_c Concentration expansion co-efficient
λ unsteadiness parameter
ρ density of fluid
λ_1 Thermal bouncy parameter
λ_2 Concentration boundary parameter
σ Stephan Boltzmann constant

References

1 Bejan A, *Adv Heat Transf*, (1982) 1.
2 Bejan A, *Int J Heat Fluid Flow*, 2 (1987) .
3 Srinivasacharya D & Bindu K H, *Ain Shams Eng J*, (2016).
4 Srinivasacharya D & Bindu K Bima, *Alexandria Eng J*, 55 (2016) 973.
5 Noghrehabadi S A M R, Pourrajab R & Ghalambaz M, *J Mech Sci Technol*, 27 (2013) 927.
6 Govindaraju M, Ganga B & Abdul H A K, *Front Heat Mass Transf*, 8 (2017).
7 Govindaraju M, Akilesh M & Abdul Hakeem A K, *J Phys Conf Ser*, 1139 (2018) 012010.
8 Afridi M I & Qasim M, *J Nanofluids*, 7 (2018) 783.
9 Abolbashari M H, Freidoonimehr N, Nazari F & Rashidi M M, *Adv Powder Technol*, 26 (2015) 542.
10 Dey D & Hazarika M, *Lat Am Appl Res - An Int J*, 50 (2020) 209.
11 Navier C L, *Mémoires l'Académie des Sci l'Institut Fr*, (1823).
12 Das K, *Heat Mass Transf Stoffuebertr*, 48 (2012) 767.
13 Zhu J, Chu P & Sui J, *Math Probl Eng*, (2018) 3094980.

14 Chauhan D S & Olkha A, *Chem Eng Commun*, 198 (2011) 1129.
15 Hayat T, Javed T & Abbas Z, *Int J Heat Mass Transf*, 51 (2008) 4528.
16 Jusoh R & Nazar R, *J Phys Conf Ser*, 1212 (2019) 012024.
17 Das K, Duari P R & Kundu P K, *J Egypt Math Soc*, 23 (2015) 435.
18 Basir M F M, Ismail F A, Amirsom N A, Latiff N A A, & Ismail A I M, *AIP Conf Proc*, 1830 (2017) 020018.
19 Mabood F & Shateyi S, *Model Simul Eng*, 2019 (2019).
20 Ali M E, *Wärme-und Stoffübertragung* , 29 (1994) 227.
21 Bejan A, *J Appl Phys* (1996).
22 Shampine L & Kierzenka J & Reichelt M, *Tutor Notes*, (2000).
23 Dey D , Hazarika M & Borah R, *Lat Am Appl Res - An Int J*, 51 (2021) 255.
24 Dey D & Chutia B, *J King Saud Univ - Eng Sci*, (2020).